

Example Find the midpoint, M, of the line segment, PQ, with end points P(3, -7) and Q(-5, 11).

Solution Use $(x_1, y_1) = (3, -7)$ and $(x_2, y_2) = (-5, 11)$.

$$\begin{aligned}\text{Use } M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{3 - 5}{2}, \frac{-7 + 11}{2} \right) \\ &= \left(\frac{-2}{2}, \frac{4}{2} \right) \\ &= (-1, 2)\end{aligned}$$

The midpoint, M, of line segment, PQ, has co-ordinates (-1, 2).

4.9 Exercise

- A** Review the method of finding the co-ordinates of the midpoint of a line segment.
- Line segment, PQ, has end points P(8, 4) and Q(2, 6).
 - Sketch the information on a diagram.
 - What are the co-ordinates of the midpoint?
 - Find the co-ordinates of the midpoint of the line segment with these end points.

(a) A(1, 6), B(9, 6)	(b) D(-3, 3), E(-9, 3)	(c) G(6, -1), H(6, -7)
(d) M(-7, 4), N(-7, -4)	(e) P(4, 4), Q(8, 4)	(f) R(3, 8), S(3, 4)
 - Find the midpoint of each line segment given by the co-ordinates of the end points.

(a) (-1, -2), (-7, 10)	(b) (6, 4), (0, 0)	(c) (5, -1), (-2, 9)
(d) (0, 0), (6, 4)	(e) (4, -5), (9, -6)	(f) (0, -4), (12, 0)
(g) (-2, 3), (3, 5)	(h) (5, 0), (-8, -3)	(i) (-7, -11), (-5, 0)
 - Find the midpoint of each of the following.

(a) AB A(5, 3), B(1, 5)	(b) CD C(-4, -5), D(2, 3)	(c) EF E(-6, 3), F(6, -7)
-------------------------	---------------------------	---------------------------
- B** Remember: When you answer a problem, make a final statement.
- A diameter of a circle has end points A(9, -4) and B(3, -2). Find the centre of the circle.
 - The end points of AB are $A(\sqrt{72}, -\sqrt{12})$ and $B(\sqrt{32}, -\sqrt{48})$. Find the midpoint.

- 7 One end point of line segment AB is $A(-2, 4)$. If the co-ordinates of the midpoint are $(-1, 7)$, find the co-ordinates of B.
- 8 If the midpoint of a segment is $(-1, -8)$ and one end point is $(7, -9)$, find the co-ordinates of the remaining end point.
- 9 $D(-5, 8)$, $E(-5, -6)$, and $F(9, 8)$ are the vertices of $\triangle DEF$.
- (a) Find the midpoint M of DE. (b) Find the midpoint N of DF.
(c) Find the length of MN. (d) Find the length of the base EF.
(e) How do the lengths of MN and EF compare?
- 10 The vertices of $\triangle ABC$ are $A(2, 8)$, $B(-2, -8)$, and $C(-14, 4)$.
P and Q are the midpoints of AB and AC.
- (a) Sketch a diagram of the given information.
(b) Use co-ordinates to show that $PQ = \frac{1}{2}BC$.
- 11 $B(-2, 16)$, $C(10, 4)$, $D(-2, -8)$, and $E(-14, 4)$ are the vertices of a square.
- (a) Show that the diagonals are equal in length.
(b) Show that the diagonals bisect each other.
- 12 If 2893 digits are used to number the pages of a book, how many pages does the book have?